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The Influence of an External Magnetic Field on the Magnetic Part of the Specific Heat of a Ferromagnetic Substance

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An equation is derived for the magnetic part of the specific heat of iron using the Weiss Molecular Field Theory. Numerical values are obtained, using a computer, for the magnetic part of the specific heat with an external magnetic field. The results show that an external magnetic field has a considerable influence on the magnetic specific heat at the Curie temperature.

1. Introduction

Recently there have been rather extensive investigations into the physical properties of a ferromagnetic material in the vicinity of the Curie temperature. However, most of these papers deal with no external magnetic field. This is especially true in the case of the specific heat. Although measurements by Potter [1] of the magneto-caloric effect of a ferromagnetic substance, which is related to the specific heat, do indicate a considerable influence at the Curie temperature when an external magnetic field is applied. It is the purpose of this paper to study the influence of such a field on the anomaly of the specific heat.

It is customary for the specific heat of a ferromagnetic material to be broken down into several parts: first, the specific heat capacity associated with lattice vibrations; second, the specific heat capacity due to the electrons; and finally, the magnetic part of the specific heat capacity. The specific heat, in the aforementioned parts, has already been investigated for the case of no external magnetic field by Hofman, Paskin, Tower, and Weiss [2] and by Braun and Kohlhaas [3]. In this paper we shall investigate the contribution of the spins to the specific heat as a function of temperature in an external magnetic field.

Atomic systems with and without atomic interactions are dealt with in statistical mechanics. In statistical mechanics the exchange interactions between the elementary particles are usually neglected. However, in the case of a ferromagnetic material the interatomic exchange energy between the atoms cannot be neglected and it is, therefore, more difficult to derive expressions for the ferromagnetic properties, as was pointed out by Domb [4]. In order to study the influence of the external magnetic field on ferromagnetic properties, we have selected the Weiss Molecular Field Theory [5]. An equation is derived for the magnetic part of the specific heat contributed by the spins for iron and also evaluated numerically for various external fields.

2. Derivation of the Magnetic Specific Heat for Iron

Using the Molecular Field Theory, Wagner [6] obtains the following expression for the average energy of a ferromagnet:

$$E = -MH - \left(\frac{\lambda}{2}\right) M^2 \tag{1}$$

where H is the external magnetic field, M is the magnetisation, and λ is the Weiss Molecular Coefficient. The quantum mechanical equation of the state of a ferromagnetic material is given by Kouvel [7] as

$$M \doteq Mg\, sB_s\,(x)\ , \tag{2}$$

where

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$$B_s(x) = \left(\frac{2s+1}{2s}\right) \coth\left[\left(\frac{2s+1}{2s}\right) x\right] - \left(\frac{1}{2s}\right) \coth\left[\left(\frac{1}{2s}\right) x\right],$$

$$x = \left(\frac{g\beta s}{kT}\right)(H + \lambda M)\cdot$$

In this equation, the terms have the following meanings: N, the number of atoms; T, the temperature; β, the Bohr magneton; $B_s(x)$, the Brillouin function; and k, the Boltzmann constant.

In the numerical calculations for iron, the g-factor will have the value of 2, and s, the spin, will be 1, as in Kittel [8] and Weiss [9]. The above values come from experimental considerations. The experimental values of the spontaneous magnetisation as a function of temperature as given by Wagner [6] or Landolt-Börnstein [10] agree well, particularly at high temperatures, with the theoretical curve for $s = 1$, calculated from the molecular field theory. As pointed out by Morrish [11] this suggests that the magnetic properties of a ferromagnet originate from the electron spin rather than from the electron orbital angular momentum, and hence, the g-factor is about 2. The best evidence that g is approximately 2 for iron comes from experiments. Wagner [6] lists the following values: from resonance experiments 2.1, from gyromagnetic experiments 1.92. The Heisenberg model [12] for ferromagnetism also suggests that the magnetic properties arise from the spin coupling rather than from the total angular momentum.

From equations 1 and 2, an equation for the magnetic specific heat for iron can be derived. If equation 1 is differentiated with respect to temperature, holding H constant, we obtain the following expression for the magnetic specific heat:

$$c_{\mathrm{m}} = -H\left(\frac{\mathrm{d}M}{\mathrm{d}T}\right) - \lambda M\left(\frac{\mathrm{d}M}{\mathrm{d}T}\right)\cdot \tag{3}$$

We shall now attempt to derive an expression for $\mathrm{d}M/\mathrm{d}T$ using equation 2. Letting

$$a = Ng\beta s,\ b = \left(\frac{2s+1}{2s}\right), \quad c = \frac{1}{2s},\ \text{and}\ r = \frac{g\beta s}{k},$$

equation 2 can be written in the form

$$M = ab\coth\left[\left(\frac{br}{T}\right)(H+\lambda M)\right] - ac\coth\left[\left(\frac{rc}{T}\right)(H+\lambda M)\right]\cdot \tag{4}$$

Differentiating equation 4 with respect to temperature and holding H constant, we obtain

$$\frac{\mathrm{d}M}{\mathrm{d}T} = ab\left(\frac{-1}{\sinh^2 x'}\right)\frac{\mathrm{d}}{\mathrm{d}T}\left[\frac{brH}{T} + \left(\frac{br\lambda M}{T}\right)\right] - ac\left(\frac{-1}{\sinh^2 x''}\right)\frac{\mathrm{d}}{\mathrm{d}T}\left[\left(\frac{cr\,H}{T}\right) + \left(\frac{cr\lambda M}{T}\right)\right] \tag{5}$$

where

$$x' = \left(\frac{rb}{T}\right)(H+\lambda M)$$

$$\text{and } x'' = \left(\frac{cr}{T}\right)(H+\lambda M\cdot$$

Substituting the values $s = 1$, $b = 3/2$, and $c = 1/2$ into equation 5 and using some further algebraic manipulation leads to

$$\frac{\mathrm{d}M}{\mathrm{d}T} = \frac{\frac{a}{2}\left[\left(\frac{-\beta H}{kT^2}\right) - \left(\frac{\lambda\beta M}{kT^2}\right)\right]\left[\left(\frac{-9}{\sinh^2 x'}\right) + \left(\frac{1}{\sinh^2 x''}\right)\right]}{1 - \left(\frac{a\lambda\beta}{2kT}\right)\left[\left(\frac{-9}{\sinh^2 x'}\right) + \left(\frac{1}{\sinh^2 x''}\right)\right]}\cdot \tag{6}$$

Kouvel [7] showed that equation 2 can be expanded in terms of x at high temperature yielding

$$\lambda = \frac{3kT_c}{8N\beta^2}$$

where T_c, the Curie temperature, is 1040° K.

By substituting the expression for λ into equation 6, substituting $\mathrm{d}M/\mathrm{d}T$ from equation 6 into equation 3, and dividing by Nk, we obtain

$$\frac{c_{\mathrm{m}}}{Nk} = \frac{\left[\left(\frac{\beta H}{kT}\right)^2 + \frac{3\beta T_{\mathrm{c}} H}{2kT^2}\left(\frac{M}{2N\beta}\right) + \left(\frac{3T_{\mathrm{c}}}{4T}\right)^2\left(\frac{M}{2N\beta}\right)^2\right]\left[\left(\frac{-9}{\sinh^2 x'}\right) + \left(\frac{1}{\sinh^2 x''}\right)\right]}{1 - \left(\frac{3T_{\mathrm{c}}}{8\,T}\right)\left[\left(\frac{-9}{\sinh^2 x'}\right) + \left(\frac{1}{\sinh^2 x''}\right)\right]}\,. \tag{7}$$

In order to get an expression for c_{m}/Nk in a constant magnetic field as a function of temperature, we must know $M/2N\beta$ as a function of temperature and an external magnetic field. $M/2N\beta$ corresponds to M/M_0 and is called the relative magnetisation. This function is given by equation 4 and the numerical values given in table I and fig. 2, have been determined by use of the *Regula falsi* method. If these values are substituted into equation 7 we obtain the magnetic part of the specific heat as a function of temperature for a constant external field. These results are shown in fig. 1 and table I.

3. Results and Conclusions

Calculations were carried out under four conditions, as follows: (i) no magnetic field; (ii) magnetic field of 10 000 Oe; (iii) magnetic field of 15 000 Oe; (iv) magnetic field of 22 500 Oe. The results of these calculations are given in fig. 1, and the numerical values in table I. Fig. 1 also gives the magnetic part of the specific heat at zero magnetic field calculated for iron by Ballensiefen and Wagner [13], who used the Weiss Molecular Field Theory, and considered only the case of no external magnetic field, with $T_{\mathrm{c}} = 1043$ and spin one. There is good agreement between those values and those calculated above, for the case of zero magnetic field. This indicates that the general expression derived for c_{m}/Nk is valid for the special case, when the external field is zero. Fig. 2 shows the relative magnetisation curves for the above-mentioned external magnetic fields, and shows a considerable influence on the relative magnetisation.

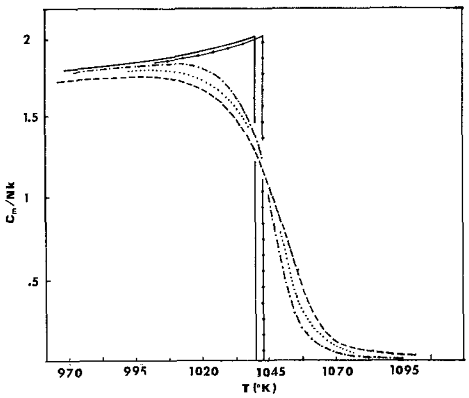

Figure 1 Curves for c_{m}/Nk versus temperature for different applied magnetic fields for iron: ——— no field; —·—·— 10 000 Oe; ········ 15 000 Oe; – – – – – 22 500 Oe; and —•—•— no field. From Ballensiefen and Wagner [13].

The numerical evaluation of the magnetic specific heat shows a very interesting behaviour pattern of a ferromagnetic material in an external magnetic field. A summary of the most important results is given below:

(i) The anomaly of the specific heat disappears. Instead of the discontinuity, we obtain a continuous curve, as fig. 1 shows.

TABLE I Numerical values for c_{m}/Nk and M/M_0*

T° K	$H = 0.00$ Oe		$H = 10\,000$ Oe		$H = 15\,000$ Oe		$H = 22\,500$ Oe	
	M/M_0	c_{m}/Nk	M/M_0	c_{m}/Nk	M/M_0	c_{m}/Nk	M/M_0	c_{m}/Nk
960	0.470	1.753	0.449	1.750	0.452	1.737	0.455	1.717
970	0.418	1.807	0.424	1.772	0.426	1.756	0.430	1.733
980	0.389	1.833	0.396	1.792	0.399	1.772	0.403	1.746
990	0.358	1.860	0.365	1.808	0.369	1.785	0.374	1.754
1000	0.323	1.887	0.332	1.819	0.337	1.791	0.343	1.752
1010	0.283	1.914	0.295	1.920	0.301	1.783	0.308	1.736
1020	0.235	1.941	0.253	1.799	0.260	1.749	0.270	1.690
1030	0.174	1.968	0.203	1.715	0.214	1.652	0.228	1.587
1040	0.050	2.000	0.131	1.308	0.161	1.390	0.181	1.360
1050			0.083	0.661	0.107	0.819	0.133	0.938
1060			0.047	0.166	0.067	0.299	0.093	0.472
1070			0.031	0.051	0,046	0.107	0.067	0.207
1080			0.023	0.021	0.034	0.046	0.051	0.973
1090			0.018	0.010	0.027	0.023	0.041	0.051

*Calculations were performed at the computer centre of the University of Cologne.

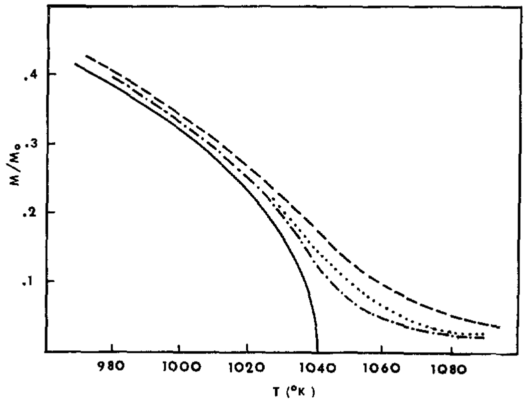

Figure 2 Curves M/M_0 versus temperature for different applied magnetic fields for iron: —— no field; —·—·— 10 000 Oe; ······· 15 000 Oe; - - - - - 22 500 Oe.

(ii) With an increase in the external magnetic field, the maximum value of the specific heat decreases.

(iii) The maximum value of the magnetic specific heat shows a shift to the left of the Curie temperature as the magnetic field increases.

These properties exhibited by a ferromagnetic substance in an external magnetic field have been confirmed by the measurements of the specific heat $c_{p,H}$ by Korn and Kohlhaas [14]. One can draw the following conclusion: that the molecular field theory gives a surprisingly good picture of the most important fundamental physical properties of a ferromagnetic material when an external magnetic field is applied.

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